FEDERAL PUBLIC SERVICE COMMISSION

COMPETITIVE EXAMINATION FOR
RECRUITMENT TO POSTS IN BS-17
UNDER THE FEDERAL GOVERNMENT, 2015

Roll Number

PURE MATHEMATICS, PAPER-II

TIME ALLOWED: THREE HOURS	MAXIMUM MARKS = 100

NOTE: (i) Attempt **ONLY FIVE** questions in all, by selecting **THREE** questions from **SECTION-I** and **TWO** questions from **SECTION-II. ALL** questions carry **EQUAL** marks.
(iii) All the parts (if any) of each Question must be attempted at one place instead of at different places.
(iv) Candidate must write Q. No. in the Answer Book in accordance with Q. No. in the Q.Paper.
(v) No Page/Space be left blank between the answers. All the blank pages of Answer Book must be crossed.
(vi) Extra attempt of any question or any part of the attempted question will not be considered.
(vii) **Use of Calculator is allowed.**

SECTION-I

Q. No. 1. (a) Use the Mean Value Theorem to show that **(10)**
$|\sin x - \sin y| \leq |x - y|$
for any real number x and y.

(b) Use Taylor's Theorem to prove that **(10)**
$\ln\sin(x+h) = \ln\sin x + h\cot x - \frac{1}{2}h^2\csc^2 x + \frac{1}{3}h^3\cot x\csc^2 x + \cdots$.

Q. No. 2. (a) Evaluate $\lim_{x\to 0}\frac{\sin x - \ln(e^x\cos x)}{x\sin x}$. **(8)**

(b) Find the equation of the asymptotes of $2xy = x^2 + 3$. **(6)**

(c) Evaluate the integral $\int_0^2 x^3(\sqrt{2x+3})\,dx$. **(6)**

Q. No. 3. (a) Verify that $f_{xy} = f_{yx}$ for the following function: **(8)**
$f(x,y) = e^{xy}\cos(bx+c)$.

(b) Find the points of relative extrema for $f(x) = \sin x\cos 2x$. **(6)**

(c) Evaluate the limit $\lim_{x\to 0}\frac{1-\cos x}{x^2}$. **(6)**

Q. No. 4. (a) Let $d: X\times X \to R$ be a metric space. Then $d': X\times X \to R$ defined by **(10)**
$d'(x,y) = \frac{d(x,y)}{1+d(x,y)}$
is also a metric.

(b) Show that an open ball in metric space X is an open set. **(5)**

(c) Show that convergent sequence in a metric space is Cauchy sequence. **(5)**

Q. No. 5. (a) Let (X,d) be a metric space, a subset A of X is dense if and only if A has non-empty intersection with any open subset of X. **(8)**

(b) Determine whether the given series converges or diverges:
$\sum_1^\infty \frac{(2n)!}{4^n}$ **(6)**

(c) Determine whether the given series converges absolutely, converges conditionally or diverges:
$\sum_1^\infty \frac{(-1)^n n!}{(2n)!}$. **(6)**

SECTION-II

Q. No. 6. (a) Use De Moivre's Theorem to evaluate $\left(\frac{\sqrt{3}-i}{\sqrt{3}+i}\right)^6$. **(10)**

(b) Evaluate $\oint_C \frac{z+2}{z} dz$, where C is the circle $z = 2e^{i\theta}$ $(0 \leq \theta \leq 2\pi)$. **(10)**

Q. No. 7. (a) Find the Laurent series that represents the function: **(10)**

$$f(z) = z^2 sin\left(\frac{1}{z^2}\right).$$

(b) Evaluate the sum of the infinite series: **(10)**

$$cos\theta - \frac{1}{2}cos2\theta + \frac{1}{3}cos3\theta - \frac{1}{4}cos4\theta + \cdots.$$

Q. No. 8. (a) Find the Fourier transform of : **(10)**

(i) $f(t) = e^{-|t|}$ (ii) $f(t) = sin\alpha t^2$

(b) Find the residue at $z = 0$ of the functions: **(10)**

(i) $f(z) = \frac{1}{z+z^2}$ (ii) $f(z) = zcos\left(\frac{1}{z}\right)$