FEDERAL PUBLIC SERVICE COMMISSION
COMPETITIVE EXAMINATION FOR
RECRUITMENT TO POSTS IN BS-17
UNDER THE FEDERAL GOVERNMENT, 2014

Roll Number

APPLIED MATHEMATICS, PAPER-II

TIME ALLOWED: THREE HOURS	MAXIMUM MARKS: 100

NOTE:(i) Candidate must write **Q.No**. in the **Answer Book** in accordance with **Q.No**. in the **Q.Paper.**
(ii) Attempt FIVE questions in all by selecting **TWO** questions from **SECTION-A, ONE** question from **SECTION-B** and **TWO** questions from **SECTION-C. ALL** questions carry **EQUAL** marks.
(iii) No Page/Space be left blank between the answers. All the blank pages of Answer Book must be crossed.
(iv) Extra attempt of any question or any part of the attempted question will not be considered.
(v) **Use of Calculator is allowed.**

SECTION-A

Q. No. 1. **(a)** Solve the initial-value problem **(10)**

$$\frac{dy}{dx} = \frac{1}{x + y^2}; \quad y(-2) = 0 .$$

(b) Initially there were 100 milligrams of a radioactive substance present. After 6 hours the mass decreased by 3%. If the rate of decay is proportional to the amount of the substance present at any time, find the amount remaining after 24 hours. **(10)**

Q. No. 2. **(a)** Solve $(x^2 + 1)y'' + xy' - y = 0$. **(10)**

(b) Obtain the partial differential equation by elimination of arbitrary functions, **(10)**
$a \sin x + b \cos y = z$ (take z as dependent variable).

Q. No. 3. **(a)** Solve the partial differential equation $u_{xx} + u_{yy} = u_t$, **(10)**

subject to the conditions
$$u(0, y, t) = u(a, y, t) = 0$$
$$u(x, 0, t) = u(x, a, t) = 0$$
and the initial condition, $u(x, y, 0) = \phi(x, y)$.

(b) Solve $r + (a + b)s + abt = xy$ by Monge's method. **(10)**

SECTION-B

Q. No. 4. **(a)** Prove that if $A_i, B_j, and\, C_k$ are three first order tensors, then their product $A_i B_j C_k \ (i, j, k = 1, 2, 3)$ is a tensor of order 3, while $A_i B_j C_k (i, j = 1, 2, 3)$ form a first order tensor. **(10)**

(b) If $A_{i_1 i_2 i_3 \cdots i_n}$ is a tensor of order *n*, then its partial derivative with respect to x_p that is $\frac{\partial}{\partial x_p} A_{i_1 i_2 i_3 \cdots i_n}$ is also a tensor of order *n+1*. **(10)**

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Q. No. 5. **(a)** Show that the transformation $\begin{bmatrix} x_1' \\ x_2' \\ x_3' \end{bmatrix} = \frac{1}{7}\begin{bmatrix} -3 & -6 & -2 \\ -2 & 3 & -6 \\ 6 & -2 & -3 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$ is orthogonal and right-handed. **(10)**

A second order tensor A_{ij} is defined in the system $Ox_1x_2x_3$ by $A_{ij} = x_i x_j \quad i, j = 1, 2, 3.$ Evaluate its components at the point P where $x_1 = 0,\ x_2 = x_3 = 1.$ Also evaluate the component A'_{11} of the tensor at P.

(b) The Christofell symbols of the second kind denoted by $\begin{Bmatrix} m \\ ij \end{Bmatrix}$ are defined **(10)**

$$\begin{Bmatrix} m \\ ij \end{Bmatrix} = g^{mk}\left[ij, k\right] \quad (i, j, k = 1, 2, \ldots n).$$

Prove that (i) $\begin{Bmatrix} m \\ ij \end{Bmatrix} = \begin{Bmatrix} m \\ ji \end{Bmatrix}$, (ii) $[ij, k] = g_{mk}\begin{Bmatrix} m \\ ij \end{Bmatrix}$,

(iii) $\dfrac{\partial g^{ij}}{\partial x^k} = -g^{im}\begin{Bmatrix} i \\ km \end{Bmatrix} - g^{jm}\begin{Bmatrix} i \\ km \end{Bmatrix}.$

SECTION-C

Q. No. 6. **(a)** Apply Newton-Raphson's method to determine a root of the equation $f(x) = \cos x - xe^x = 0$ such that $\left|f(x^*)\right| < 10^{-8}$, where x^* is the approximation to the root. **(10)**

(b) Consider the system of the equations **(10)**

$$2x_1 - x_2 + 0x_3 = 7$$
$$-x_1 + 2x_2 - x_3 = 1,$$
$$0x_1 - x_2 + 2x_3 = 1$$

Solve the system by using Gauss-Seidel iterative method and perform three iterations.

Q. No. 7. **(a)** Use the trapezoidal and Simpson's rules to estimate the integral **(10)**

$$\int_1^3 f(x)dx = \int_1^3 (x^3 - 2x^2 + 7x - 5)dx \ .$$

(b) Find the approximate root of the equation $f(x) = 2x^3 + x - 2 = 0$. **(10)**

Q. No. 8. **(a)** Find a 5th degree polynomial which passes through the 6 points given below. **(10)**

x	1.0	2.0	4.0	5.0	7.0	8.0
$f(x)$	-9	-41	-189	-173	9	523

(b) Determine the optimal solution graphically to the linear programming problem, **(10)**

$$\begin{aligned} &\textit{Minimize} \quad z = 3x_1 + 6x_2 \\ &\textit{subject to} \quad 4x_1 + x_2 \ge 20 \\ &\qquad x_1 + x_2 \le 20 \\ &\qquad x_1 + x_2 \ge 10 \\ &\qquad x_1, x_2 \ge 0 \end{aligned}$$

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